Received: *July 26, 1999*, Accepted: *November 12, 1999*

Supersymmetric baryonic branes

Joaquim Gomis and Joan Simón

Departament ECM, Facultat de Física
Universitat de Barcelona, and
Institut de Física d'Altes Energies
Diagonal 647, E-08028 Barcelona, Spain
E-mail: gomis@ecm.ub.es, jsimon@ecm.ub.es

Alfonso V. Ramallo

Departamento de Física de Partículas, Universidad de Santiago
E-15706 Santiago de Compostela, Spain
E-mail: alfonso@fpaxp1.usc.es

Paul K. Townsend

DAMTP, University of Cambridge
Silver Street, Cambridge CB3 9EW, UK
E-mail: p.k.townsend@damtp.cam.ac.uk

Abstract: We derive an energy bound for a 'baryonic' D5-brane probe in the $AdS_5 \times S^5$ background near the horizon of N D3-branes. Configurations saturating the bound are shown to be 1/4 supersymmetric S^5-wrapped D5-branes, with a total Born-Infeld charge N. Previous results are recovered as a special case. We derive a similar energy bound for a 'baryonic' M5-brane probe in the background of N M5-branes. Configurations saturating the bound are again 1/4 supersymmetric and, in the $AdS_7 \times S^4$ near-horizon limit, provide a worldvolume realization of the 'baryon string' vertex of the (2,0)-supersymmetric six-dimensional conformal field theory on coincident M5-branes. For the full M5-background we find a worldvolume realization of the Hanany-Witten effect in M-theory.

Keywords: M-Theory, D-branes, Brane Dynamics in Gauge Theories, Supersymmetry and Duality.

JHEP11(1999)019

Contents

JHEP11(1999)019

1. Introduction

Many aspects of the physics of intersecting branes of M-theory or superstring theory can be understood in terms of the worldvolume field theory of a ‘probe’ (or ‘test’) brane in the supergravity background of other branes. One attractive feature of this approach is that many such supergravity backgrounds have non-singular horizons near which they are asymptotic to Kaluza-Klein vacua with an anti-de Sitter (AdS) factor [1]; this allows one to make contact with the physics of supersymmetric conformal field theories (CFT) via the AdS/CFT correspondence [2]. A nice example is provided by a closed D5-brane ‘surrounding’ N D3-branes (such that the D3-branes thread the D5-branes). The Hanany-Witten (HW) effect [3] implies that each of the N D3-branes must be connected to the D5-brane by a (‘fundamental’) IIB string. It follows that the D5-brane cannot contract to a point but must instead contract to some lowest energy configuration involving the D3-branes and N IIB strings. In the limit of large N this configuration can be investigated by replacing the D3-branes by the corresponding supergravity background. As the D5-brane is allowed to contract (quasi-statically) it enters the near-horizon region of the background for which the geometry is $AdS_5 \times S^5$. At this point the D5-brane is wrapped on the S^5 factor and the N strings emanating from it can be considering as having their endpoints on the AdS_5 boundary. This configuration was interpreted in [5, 6] as a ‘baryon-vertex’ of $\mathcal{N} = 4$ $D = 4$ super-Yang-Mills (SYM) theory.

Although the above description of the baryon vertex involves IIB strings in addition to a D5-brane, it is also possible to view it as a single D5-brane carrying N units of Born-Infeld (BI) charge [7, 8]. If one assumes that all the BI charge is concentrated at one point, which may be taken to be a polar point, then one would expect the minimum energy configuration to be one that preserves the SO(5) rotational invariance about this point. Such a configuration is determined by giving the radial position of the D5-brane in AdS_5 as a function $r(\theta)$ of the co-latitude angle θ on S^5. One might also hope that this minimal energy configuration would preserve some fraction of the D5-brane's worldvolume supersymmetry. A condition on the function $r(\theta)$ for preservation of supersymmetry was found by Imamura [7]. An explicit solution of this condition, which was shown to preserve 1/4 supersymmetry, was then found by Callan et al. [8]. It was subsequently shown that this solution saturates an energy bound and hence minimises the energy for a fixed value of a topological charge [9].

One might think that the restriction to SO(5) invariant configurations is not merely a convenience but rather a necessity, either for supersymmetry or for minimal energy. Since the equivalence of these two conditions may not be considered a priori obvious we shall concentrate for the moment on the minimal energy condition. The minimal energy configuration with a single pointlike singularity has been interpreted as the common endpoint of N parallel strings [8]. For this interpretation to be valid it must be possible, *given sufficient energy*, to move the strings apart. In particular, it must be possible to move their endpoints on the D5-brane to any point on the 5-sphere such that r becomes a function of all five angular coordinates of the 5-sphere with N point singularities of unit charge. It is not guaranteed that a given configuration of this sort will solve the D5-brane field equations but every such configuration is a possible initial configuration satisfying the basic requirement that the D5-brane carries a total of N units of BI charge. As such it must relax to some minimal energy configuration with the same boundary conditions at the N singular points. We shall call these minimal energy configurations 'baryonic branes'. The issue that we wish to address here is whether the solution of [8] is the only baryonic D5-brane or whether it is one of a class of minimal energy solutions.

We shall attack this problem on several fronts. First, we generalize the energy bound of [9] to allow for arbitrary positions of the N singularities of the radial function on the 5-sphere. The energy is shown to be bounded below by the total charge, *whose functional form is independent of the positions of the singularities*. The conditions under which this bound is saturated are then shown to be *precisely those required for preservation of 1/4 supersymmetry*. This result generalizes the supersymmetry condition of [7]. Our method, which makes use of the κ-symmetry transformations of the D5-brane [10], differs from that used in [7]. As for any super-brane action in any background compatible with κ-symmetry, the κ-symmetry transforma-

JHEP11(1999)019

tions involve a matrix Γ_κ, characteristic of the background and type of brane. The number of supersymmetries preserved by the combined background/brane system is the number of linearly-independent solutions of [11]

$$\Gamma_\kappa \chi = \chi\,, \tag{1.1}$$

where χ is an arbitrary linear combination of Killing spinors of the background.[1]

Thus, we shall uncover no evidence from either supersymmetry or Bogomol'nyi-type bounds that the SO(5) invariant solution of [8] is the unique baryonic brane configuration of either minimal energy or 1/4 supersymmetry. Within the subclass of D5-brane configurations specified by giving N pointlike singularities of unit charge at specified points on S^5 it is natural to suppose that the one of minimal energy saturates the energy bound because there is no apparent obstacle that prevents this. We shall derive an equation for the function r that is both necessary and sufficient for both saturation of the bound and 1/4 supersymmetry, so the issue of whether the bound is saturated boils down to the existence or otherwise of solutions to this equation subject to specified boundary conditions. For a single point singularity the equation is solved by the SO(5) invariant configuration of [8]. We have found a larger class of solutions, of no particular symmetry, that depend on all five angular coordinates of S^5, but the function r has branch cuts that make its interpretation difficult. We have not found any solutions with isolated point singularities other than the SO(5) invariant one. This may be because they do not exist, but it could equally well be because they are hard to find. This remains an open question.

Another purpose of this paper is to generalize these results for the baryonic D5-brane to the baryonic M5-brane. To see what this entails, we recall that the HW effect has an M-theory analogue [3, 4]. This effect implies that a closed M5-brane with one direction parallel to N other M5-branes, and otherwise lying on a 4-sphere at fixed radius from them, must be connected to each of the enclosed M5-branes by an M2-brane, with one dimension of the M2-brane in the common direction of all M5-branes. Let us replace the N M5-branes by a supergravity M5-brane and consider the remaining one as a probe. Near the horizon of the supergravity M5-brane the geometry is that of $AdS_7 \times S^4$ and if the probe is allowed to contract it will end up as an M5-brane wrapped on the 4-sphere. This M5-brane is the 'baryon string' vertex of the (2,0) conformal field theory on the AdS_7 boundary [6, 12]. It is connected to the AdS_7 boundary by N M2-branes; from the AdS_7 perspective these N M2-branes appear to meet in a string-like junction at some point in the AdS_7 interior.

Here we will show that it is posible to view this configuration of M5-brane and N M2-branes as a single M5-brane carrying N units of the charge associated with the self-dual closed worldvolume 3-form. By analogy with the D5-brane case, we

[1]In all previous applications of this formula it has been found possible to replace χ by a constrained but *constant* spinor ϵ, but this turns out not to be possible for the cases considered here.

JHEP11(1999)019

will call this a ‘baryonic M5-brane’. A baryonic M5-brane wraps the 4-sphere of $AdS_7 \times S^4$ with singularities to take account of the non-zero charges. We derive an energy bound satisfied by such configurations and show that the configurations that saturate it preserve 1/4 of the bulk supersymmetry. We thereby find an equation for a function on S^4 that is both necessary and sufficient for either 1/4 supersymmetry and minimal energy. The general solution of this equation will depend on all four angular coordinates of S^4. We have found explicit solutions of this type, although they again involve branch cuts and do not have an obvious baryonic-brane interpretation, except in the subcase of configurations that are invariant under the SO(4) subgroup of the SO(5) isometry group of the 4-sphere. These are specified by a radial function of a single angular coordinate. The requirement of 1/4 supersymmetry then reduces to a differential equation for this radial function which turns out to be the same as one recently found, and solved, for a baryonic D4-brane in a D4-brane background [13, 14].

We shall begin with an analysis of the baryonic D5-brane in the background of N D3-branes and then carry out a similar analysis for the baryonic M5-brane in the background of N M5-branes. The connection with baryon vertices involves a replacement of these backgrounds by their near-horizon limits but our methods are general and apply both to the full backgrounds and their near-horizon limits. The physics of the full background case is, however, rather different as one cannot expect to find a *static* D5-brane wrapped around N D3-branes at an arbitrary distance from them. One instead finds ‘partially-wrapped’ configurations that provide a worldvolume realization of the HW effect [8]. These results have since been extended to all Dp-branes [14]. Because our analysis of supersymmetry applies to the full backgrounds we are able to establish the partial supersymmetry of these D-brane configurations.

In the M5-brane case neither the near-horizon nor the full M5-brane background has been previously considered. Our results for the near-horizon case are as summarized above. In the full background we again cannot expect to find static S^4-wrapped solutions but we can, and do, find ’partially-wrapped’ M5-branes that still preserve 1/4 supersymmetry. These results provide us with a detailed worldvolume description of the HW effect in M-theory.

2. Baryonic D5-brane

The D3-brane solution of IIB supergravity has all fields vanishing except the dilaton, which is constant, and the 10-metric and self-dual 5-form field strength $G_{(5)}$. For N coincident D3-branes at the origin, these take the form

$$ds^2_{(10)} = U^{-1/2}\, ds^2(\mathbb{E}^{(1,3)}) + U^{1/2}\left[dr^2 + r^2 d\Omega_5^2\right], \tag{2.1}$$

$$G_{(5)} = 4R^4\left[\omega_{(5)} + \star\omega_5\right], \tag{2.2}$$

JHEP11(1999)019

where $d\Omega_5^2$ is the SO(6)-invariant metric on the unit 5-sphere, ω_5 is its volume 5-form and $\star\omega_5$ its Hodge dual. The function U is

$$U = a + \left(\frac{R}{r}\right)^4 , \qquad R^4 = 4\pi g_s N(\alpha')^2 , \tag{2.3}$$

where g_s is the string coupling constant and α' the inverse IIB string tension. We set $a = 1$ for the full D3-brane solution and $a = 0$ for its near-horizon limit.

We now put a probe D5-brane of unit tension into this background such that it wraps the 5-sphere. Let $\xi^\mu = (t, \theta^i)$ be the worldvolume coordinates, so that θ^i $(i = 1, \ldots, 5)$ are coordinates for the worldspace 5-sphere. The bosonic part of the D5-brane action is

$$S = -\int_{R\times S^5} d^6\xi \left[\sqrt{-\det(h+F)} - V \wedge G_{(5)}\right] , \tag{2.4}$$

where h is the induced worldvolume metric and $F = dV$ is the Born-Infeld U(1) field strength. Because of the coupling of V to the background 5-form field strength $G_{(5)}$, the D5-brane must carry BI electric charge [5]. It follows that we cannot take F to vanish but we can assume vanishing magnetic charge, in which case F_{0i} are the only non-zero components of F. Following [7, 8, 9], we shall now proceed as if we were seeking a worldvolume soliton on the D5-brane, represented by a (radial) deformation of the D5-brane in the direction separating it from the backgound D3-branes. Such a configuration can be represented as in the array on the right.

D3:	1 2 3 _ _ _ _ _ _	background
D5:	_ _ _ 4 5 6 7 8 _	probe
F1:	_ _ _ _ _ _ _ _ 9	soliton

Configuration 1: A IIA string intersecting a D5-brane probe in a D3-brane background. The 9-direction is radially transverse to the D5-brane.

In order to analyze this configuration we choose angular coordinates Θ^i $(i = 1, \ldots, 5)$ on the 5-sphere at radius r. We then fix the worldvolume diffeomorphisms by choice of the 'static gauge'

$$X^0 = t , \qquad \Theta^i = \theta^i . \tag{2.5}$$

It now follows from the hamiltonian formulation of the super D-brane in a general background [15] that the hamiltonian density for static configurations is given by

$$\mathcal{H}^2 = U^{-1/} \left[\tilde{E}^i \tilde{E}^j g_{ij} + \det g\right] , \tag{2.6}$$

where g is the induced *worldspace* metric and $\tilde{E}^i$ is a 'covariantized' electric field density related to F_{0i} by

$$(\det g)\, F_{0i} = \sqrt{-\det(h+F)}\, \tilde{E}^j g_{ij} . \tag{2.7}$$

JHEP11(1999)019

For the background/probe configuration considered here, and in the static gauge, the Gauss law constraint is

$$\partial_i \tilde{E}^i = -4\, R^4 \sqrt{\det \bar{g}}\,, \tag{2.8}$$

where $\bar{g}_{ij}$ is the SO(6)-invariant metric on the unit 5-sphere. The source term, due to the non-vanishing 5-form field strength of the D3-brane background, confirms the statement made above that the D5-brane carries a non-zero electric charge.

Since we are considering only radial deformations of the D5-brane we may set

$$X^1 = X^2 = X^3 = 0\,, \tag{2.9}$$

which leaves the radial function r as the only 'active scalar'. The worldvolume metric for *static* configurations is now

$$h_{\mu\nu} = \begin{pmatrix} -U^{-1/2} & 0 \\ 0 & g_{ij} \end{pmatrix}, \tag{2.10}$$

where

$$g_{ij} = U^{1/2} \left(r^2 \bar{g}_{ij} + \partial_i r \partial_j r \right) . \tag{2.11}$$

The relation (2.7) now becomes

$$\Delta_5^2 \left(r^2 + \bar{g}^{ij} \partial_i r \partial_j r \right) F_{0i} = U \left[\det \left(r^2 \bar{g}_{ij} + \partial_i r \partial_j r - F_{0i} F_{0j} \right) \right]^{1/2} \left(r^2 \bar{g}_{ij} + \partial_i r \partial_j r \right) \tilde{E}^j\,, \tag{2.12}$$

where

$$\Delta_5 = U\, r^4 \sqrt{\det \bar{g}}\,. \tag{2.13}$$

This implies

$$\tilde{E}^i \;=\; U^{1/4}\, \frac{\sqrt{\det g}}{\sqrt{1 \,-\, U^{1/2}\, g^{mn}\, F_{0m}\, F_{0n}}}\; g^{ij}\, F_{0j}\,. \tag{2.14}$$

2.1 Energy bound

We now proceed to derive an energy bound for baryonic D5-branes. We shall do this, following [16, 9], by expressing $\mathcal{H}^2$ as a sum of squares. This can be done in more than one way and one gets a meaningful bound on the energy only if the quantity by which $\mathcal{H}$ is bounded is independent of the quantities neglected in arriving at the bound. This will be the case if $\mathcal{H}$ is bounded by the integrand of a topological charge, and this requirement serves to resolve ambiguities. The result we obtain by this method will be confirmed in the next subsection by an analysis of supersymmetry.

We begin by rewriting the energy density (2.6) as

$$\mathcal{H}^2 = r^2 \tilde{E}^i \tilde{E}^j \bar{g}_{ij} + (\tilde{E}^i \partial_i r)^2 + \, (U r^4)^2 r^2 \det \bar{g} + (U r^4)^2 \det \bar{g}\, \bar{g}^{ij} \partial_i r \partial_j r\,. \tag{2.15}$$

JHEP11(1999)019

This expression is manifestly SO(6) invariant but minimal energy configurations can be at most SO(5) invariant. We shall later wish to exploit this fact, so we write the 5-sphere metric as

$$ds^2 = d\theta^2 + \sin^2\theta \, d\Omega_4^2 \,, \tag{2.16}$$

where $d\Omega_4^2$ is the SO(5) invariant metric on the 4-sphere, which we take to have coordinates $\theta^{\hat\imath}$. We may now rewrite (2.15) as

$$\begin{aligned} \mathcal{H}^2 \;=\; & \Delta_5^2\left(r^2 + (r')^2 + \bar g^{\hat\imath\hat\jmath}\partial_{\hat\imath} r\,\partial_{\hat\jmath} r\right) + \left(\tilde E^\theta r'\right)^2 + \left(\tilde E^{\hat\imath}\partial_{\hat\imath} r\right)^2 + \\ & + 2\tilde E^\theta r' \tilde E^{\hat\imath}\partial_{\hat\imath} r + \left(\tilde E^\theta r\right)^2 + r^2 \tilde E^{\hat\imath}\tilde E^{\hat\jmath}\bar g_{\hat\imath\hat\jmath} \,, \end{aligned} \tag{2.17}$$

where the primes indicate derivatives with respect to θ. Note that $\bar g^{\hat\imath\hat\jmath}$ are the $\hat\imath\hat\jmath$ components of the inverse S^5 metric $\bar g^{ij}$.

We can further rewrite (2.17) as

$$\mathcal{H}^2 = \mathcal{Z}_5^2 + \left[\Delta_5\,(r\cos\theta)' - \tilde E^i \partial_i\,(r\sin\theta)\right]^2 + \left|\Delta_5 \bar g^{\hat\imath\hat\jmath}\partial_{\hat\jmath} r - r\,\tilde E^{\hat\imath}\right|^2 \,, \tag{2.18}$$

where $||^2$ indicates contraction with $g_{\hat\imath\hat\jmath}$, and

$$\mathcal{Z}_5 = \Delta_5\,(r\sin\theta)' + \tilde E^i \partial_i\,(r\cos\theta)\,. \tag{2.19}$$

Using the Gauss' law (2.8), one can show that $\mathcal{Z}_5 = \partial_i \mathcal{Z}_5^i$ where $\vec{\mathcal{Z}}_5$ has components

$$\begin{aligned} \mathcal{Z}_5^\theta \;&=\; \tilde E^\theta\, r\cos\theta + \sqrt{\det \bar g}\,\sin\theta\left(a\,\frac{r^5}{5} + r\,R^4\right), \\ \mathcal{Z}_5^{\hat\imath} \;&=\; \tilde E^{\hat\imath}\, r\cos\theta\,. \end{aligned} \tag{2.20}$$

From (2.18), and the fact that $\mathcal{Z}_5$ is a divergence, we deduce the bound

$$\mathcal{H} \geq |\mathcal{Z}_5|\,, \tag{2.21}$$

with equality when

$$\tilde E^{\hat\imath} \;=\; \Delta_5 \frac{\bar g^{\hat\imath\hat\jmath}\partial_{\hat\jmath} r}{r}\,, \tag{2.22}$$

$$\tilde E^\theta \;=\; \frac{\Delta_5}{(r\sin\theta)'}\left((r\cos\theta)' - \frac{\bar g^{\hat\imath\hat\jmath}\partial_{\hat\imath} r\,\partial_{\hat\jmath}\,(r\sin\theta)}{r}\right). \tag{2.23}$$

When combined with the Gauss law (2.8), these conditions yield the equation

$$\partial_{\hat\imath}\left(\Delta_5 \bar g^{\hat\imath\hat\jmath}\frac{\partial_{\hat\jmath} r}{r}\right) + \partial_\theta\left[\frac{\Delta_5}{(r\sin\theta)'}\left((r\cos\theta)' - \bar g^{\hat\imath\hat\jmath}\partial_{\hat\imath} r\,\partial_{\hat\jmath} r\,\frac{\sin\theta}{r}\right)\right] = -4R^4\sqrt{\det \bar g}\,. \tag{2.24}$$

We shall analyse this equation in more detail later, after rederiving it as a condition for preservation of 1/4 supersymmetry.

JHEP11(1999)019

The first-order equations (2.22) and (2.23) have been obtained for an arbitrary value of the coefficient a. As we are mostly interested in describing the baryonic vertex in the near-horizon region of the background, we shall take $a = 0$ in the remainder of this subsection (for an analysis of the $a = 1$ case see refs. [8, 14]). From the bound (2.21) we deduce the bounds

$$H \geq \int d^5\sigma |\mathcal{Z}_5| \geq \left| \int d^5\sigma \, \mathcal{Z}_5 \right| \tag{2.25}$$

on the total energy H. The first inequality is saturated under the conditions just obtained. The second inequality will be saturated too if $\mathcal{Z}_5$ does not change sign within the integration region. If the D5-brane is to describe a baryonic brane we must take this region to be the 5-sphere with some number of singular points removed. For the moment we shall simply assume that the second inequality *is* saturated when the first one is, in which case the total energy equals the 4-form dual of $\vec{\mathcal{Z}}_5$ integrated over the 4-boundary of the D5-brane. In other words

$$H = \lim_{\delta \to 0} \sum_k \int_{B_k} d\vec{S} \cdot \vec{\mathcal{Z}} \,, \tag{2.26}$$

where B_k is a 4-ball of radius δ with the k'th singular point as its centre. The k'th term in the sum can be viewed as the energy of the IIB string(s) attached to the k'th singular point. Since we will not be able to provide an explicit solution of (2.24) with these boundary conditions unless $k = 1$ we should stress that the right hand side of (2.26) is still a lower *bound* on H *even if the bound cannot be saturated.*

To make contact with previous work we now specialize to SO(5) invariant configurations. In this case $\tilde{E}^{\hat{\imath}} = 0$ and

$$\tilde{E}^\theta = \sqrt{\det g^{(4)}}\, E(\theta) \,, \tag{2.27}$$

where E is a function only of θ. Similarly, r is now a function only of θ. It will also be convenient to set

$$\Delta_5 = \sqrt{\det g^{(4)}}\, \Delta \tag{2.28}$$

since $\Delta = R^4 \sin^4\theta$ is also a function only of θ. For such SO(5)-invariant configurations the first BPS condition (2.22) is trivially satisfied while (2.23) reduces to

$$\frac{r'}{r} = \frac{\Delta \sin\theta + E \cos\theta}{\Delta \cos\theta - E \sin\theta} \,. \tag{2.29}$$

This equation coincides with the BPS equation found previously for SO(5)-invariant configurations [7, 8, 9]. For these configurations the Gauss law (2.8) becomes the ordinary first order differential equation

$$E' = -4R^4 \sin^4\theta \,, \tag{2.30}$$

JHEP11(1999)019

the solution of which is given by [8]

$$E = \frac{1}{2} R^4 \left[3 \left(\nu\pi - \theta \right) + 3 \sin\theta \cos\theta + 2 \sin^3\theta \cos\theta \right] , \tag{2.31}$$

where ν is an integration constant. As pointed out in [8], the symmetry of the problem allows us to restrict ν to lie in the interval $[0, 1]$.

We now return to consider whether the second inequality of (2.25) is saturated when the first one is. We shall restrict our discussion of this point to the SO(5)-invariant case, for which we can write

$$\mathcal{Z}_5 = \sqrt{\det g^{(4)}}\, \mathcal{Z}(\theta) , \tag{2.32}$$

where $\mathcal{Z}$ is a function only of θ. Using (2.29) we see that

$$\mathcal{Z} = r \, \frac{\left(\Delta\cos\theta - E\sin\theta\right)^2 + \left(\Delta\sin\theta + E\cos\theta\right)^2}{\left(\Delta\cos\theta - E\sin\theta\right)} , \tag{2.33}$$

the sign of which is determined by the sign of the denominator. It follows that $\mathcal{Z}$ will not change sign as long as it has no singularities within the integration region $\theta \in [0, \pi]$ (except, possibly, at the endpoints $\theta = 0, \pi$). Now

$$\Delta\cos\theta - E\sin\theta = \frac{3}{2} R^4 \sin\theta \, \eta(\theta) , \tag{2.34}$$

where

$$\eta(\theta) \equiv \theta - \nu\pi - \sin\theta\cos\theta . \tag{2.35}$$

We see from (2.34) that the denominator of the expression (2.33) for $\mathcal{Z}$ vanishes at the endpoints $\theta = 0, \pi$ but is otherwise positive provided that $\eta(\theta)$ is positive. For $\nu \in [0, 1]$ this condition is satisfied only for $\nu = 0$. In this case (2.31) becomes

$$E = \frac{1}{2} R^4 \left[3 \left(\sin\theta\cos\theta - \theta \right) + 2 \sin^3\theta \cos\theta \right] . \tag{2.36}$$

Using this result for $E(\theta)$ in (2.29) we find a first order equation for $r(\theta)$ for which the solution is [8]

$$r = r_0 \left(\frac{6}{5} \right)^{1/3} (\mathrm{cosec}\,\theta)(\theta - \sin\theta\cos\theta)^{1/3} , \tag{2.37}$$

where r_0 is the value of r at $\theta = 0$. It was shown in [8] that this configuration corresponds to N fundamental strings attached to the D5-brane at the point $\theta = \pi$, where $r(\theta)$ diverges. One may verify that this is a solution of the second order equation

$$\left[\sin^4\theta \, \frac{r'\cos\theta - r\sin\theta}{r'\sin\theta + r\cos\theta} \right]' = -4\sin^4\theta , \tag{2.38}$$

found by combining (2.30) with (2.29).

JHEP11(1999)019

The solutions of the first-order equation (2.29) for $\nu \neq 0$ have been obtained in [8]. In these solutions the range of values of θ for which the solution makes sense does not coincide with the interval $[0, \pi]$ because the D5-brane does not wrap completely the 5-sphere. In fact, when $\nu \neq 0$ the D5-brane reaches the point $r = 0$, and can therefore be interpreted as intersecting the D3-brane. Because the D5-brane only partially wraps the 5-sphere it captures only part of the five-form flux. This is consistent with the fact that the tensions of the D5-brane spikes of these partially-wrapped solutions correspond to a number of strings less than N. In their study of baryonic multiquark states with $k < N$ quarks in $\mathcal{N} = 4$ $D = 4$ super Yang-Mills theory, the authors of [17] consider configurations in which k strings connect the D5-brane to $r = \infty$ while the remaining $N - k$ strings connect it to $r = 0$. It is tempting to think that these configurations could be related to the $\nu \neq 0$ solutions of the BPS equation.

2.2 Supersymmetry

We will now show that configurations saturating the bound (2.21) preserve 1/4 of the bulk supersymmetry. Our starting point is the equation (1.1). The matrix Γ_κ is given in [10] as a (finite) expansion in powers of the BI field strength. Because we assume a purely electric field strength, this expansion terminates at linear order and the resulting matrix is

$$\Gamma_\kappa = \frac{1}{6!} \frac{1}{\sqrt{-\det(h+F)}} \epsilon^{\mu_1 \cdots \mu_6} \left[\Gamma_{\mu_1 \dots \mu_6} \sigma_1 + 15 \, F_{\mu_1 \mu_2} \Gamma_{\mu_3 \dots \mu_6} (i\sigma_2) \right] , \tag{2.39}$$

where F_{0i} are the only non-vanishing components of the Born-Infeld 2-form. The matrices $\Gamma_{\mu\nu\dots}$ are antisymmetrized products of the (reducible) induced worldvolume gamma matrices

$$\Gamma_\mu = \partial_\mu X^m E_m{}^{\underline{m}} \Gamma_{\underline{m}} , \tag{2.40}$$

where $\Gamma_{\underline{m}}$ are the constant $D = 10$ Dirac matrices and $E_m{}^{\underline{m}}$ is the spacetime vielbein. In the static gauge and for static configurations with $X^1 = X^2 = X^3 = 0$ we have

$$\begin{aligned} \Gamma_0 &= U^{-1/4} \Gamma_{\underline{0}} , \\ \Gamma_i &= U^{1/4} \, r \gamma_i + U^{1/4} \, \partial_i r \Gamma_{\underline{r}} . \end{aligned} \tag{2.41}$$

Here we have introduced the matrix

$$\gamma_i = e_i{}^{\underline{i}} \Gamma_{\underline{i}} , \tag{2.42}$$

where $e_i{}^{\underline{i}}$ is the S^5 fünfbein. Thus

$$\{\gamma_i, \gamma_j\} = 2 \bar{g}_{ij} . \tag{2.43}$$

The Killing spinors of the D3-brane background take the form

$$\chi = U^{-1/8} \epsilon , \tag{2.44}$$

JHEP11(1999)019

where ϵ is a covariantly constant spinor on $\mathbb{E}^{(1,3)} \times \mathbb{E}^6$ subject to a 1/2 supersymmetry breaking condition imposed by the background, which we discuss below. The supersymmetry preservation condition (1.1) is thus reduced to

$$\Gamma_\kappa \epsilon = \epsilon\,. \tag{2.45}$$

Although ϵ is constant in cartesian coordinates on $\mathbb{E}^{(1,3)} \times \mathbb{E}^6$, it is not constant in the polar coordinates for $\mathbb{E}^6$ that we are using here. In polar coordinates a covariantly constant spinor is independent of the radial variable and hence covariantly constant on S^5. Covariantly constant spinors on S^n were constructed explicitly in [18] for a parameterisation in which the (diagonal) metric is found by iteration of $ds_n^2 = d\theta_n^2 + \sin^2\theta_n ds_{n-1}^2$. The result can expressed in terms of the n angles $\theta^i = (\theta, \theta^{\hat{\imath}})$ and the antisymmetrixed products of pairs of the constant $D = 10$ Dirac matrices matrices $\Gamma_{\underline{i}} = (\Gamma_{\underline{\theta}}, \Gamma_{\underline{\hat{\imath}}})$. For $n = 5$ we have, with the understanding that $\Gamma_{\underline{\hat{5}}} \equiv \Gamma_{\underline{\theta}}$,

$$\epsilon = e^{\frac{\theta}{2}\Gamma_{\underline{r\theta}}} \prod_{\hat{\jmath}=1}^{4} e^{-\frac{\theta^{\hat{\jmath}}}{2}\Gamma_{\underline{\hat{\jmath}\hat{\jmath}+1}}}\, \epsilon_0\,, \tag{2.46}$$

where ϵ_0 is a constant spinor subject to

$$\Gamma_{\underline{0123}}(\,i\,\sigma_2)\epsilon_0 = \epsilon_0\,, \tag{2.47}$$

which is the 1/2 supersymmetry breaking condition associated with the D3-brane background. There are additional Killing spinors in the near-horizon limit but the corresponding background supersymmetries are not preserved by the baryonic D5-brane probe and can therefore be ignored.

Using the expression (2.39) for Γ_κ we find, after some simplification, that the supersymmetry preservation condition (2.45) reduces to

$$\begin{aligned} U\sqrt{\det\left[r^2\bar{g}_{ij} + \partial_i r\partial_j r - F_{0i}F_{0j}\right]}\epsilon = \Big[& Ur^5\sqrt{\det\bar{g}}\,\Gamma_{\underline{0}}\gamma_*\sigma_1 - \\ & - Ur^3\sqrt{\det\bar{g}}\,F_{0j}\partial_i r\gamma^{ij}\gamma_*\Gamma_{\underline{r}}(i\sigma_2) + \\ & + Ur^4\sqrt{\det\bar{g}}\gamma^i\gamma_*\left(F_{0i}(i\sigma_2) + \partial_i r\Gamma_{\underline{0r}}\sigma_1\right)\Big]\epsilon\,, \end{aligned} \tag{2.48}$$

where $\gamma^i = \bar{g}^{ij}\gamma_j$ and

$$\gamma_* = \Gamma_{\underline{45678}}\,. \tag{2.49}$$

We shall seek solutions of this equation for ϵ_0 satisfying the constraints

$$\Gamma_{\underline{0}}\gamma_*\sigma_1\epsilon_0 = \epsilon_0\,, \tag{2.50}$$

which is expected from the *local* preservation of 1/2 supersymmetry by the D5-brane, and

$$\Gamma_{\underline{0r}}\,\sigma_3\epsilon_0 = \epsilon_0\,, \tag{2.51}$$

JHEP11(1999)019

which is the condition associated with a IIB string in the radial direction. These relations imply

$$\begin{aligned}\Gamma_{\underline{0r}}\,\sigma_3\,\epsilon &= [\cos\theta - \sin\theta\,\Gamma_{\underline{r\theta}}]\,\epsilon\,,\\ \Gamma_{\underline{0}}\,\gamma_*\,\sigma_1\,\epsilon &= [\cos\theta - \sin\theta\,\Gamma_{\underline{r\theta}}]\,\epsilon\,.\end{aligned}\tag{2.52}$$

Other useful conditions that also follow are:

$$\begin{aligned}\Gamma_{\underline{i}}\,\gamma_*\,(i\sigma_2)\,\epsilon &= -\Gamma_{\underline{ri}}\,\epsilon\,,\\ \Gamma_{\underline{i}}\,\gamma_*\,\Gamma_{\underline{0r}}\,\sigma_1\,\epsilon &= \Gamma_{\underline{ri}}\,e^{-\theta\Gamma_{\underline{r\theta}}}\,\epsilon\,,\\ \gamma_*\,\Gamma_{\underline{r}}\,(i\sigma_2)\,\epsilon &= -\epsilon\,.\end{aligned}\tag{2.53}$$

Using the relations (2.52) and (2.53), one can rewrite the right hand side of (2.48) as

$$\begin{aligned}\Delta_5\,\Big[\,&(r\sin\theta)' + \Gamma_{\underline{r\theta}}\,((r\cos\theta)' - F_{0\theta}) + \Gamma_{\underline{r}}\gamma^{\hat{\imath}}\,(\partial_{\hat{\imath}}r\cos\theta - F_{0\hat{\imath}}) +\\ &+\gamma^{\hat{\imath}\hat{\jmath}}\,\frac{1}{r}\,(\partial_{\hat{\imath}}rF_{0\hat{\jmath}} - \partial_{\hat{\jmath}}rF_{0\hat{\imath}}) + \gamma^{\hat{\imath}}\Gamma_{\underline{\theta}}\,\frac{1}{r}\,(\partial_{\hat{\imath}}rF_{0\theta} - r'F_{0\hat{\imath}} + r\partial_{\hat{\imath}}r\sin\theta)\Big]\,.\end{aligned}\tag{2.54}$$

Requiring the coefficients of $\Gamma_{\underline{r\theta}}$ and $\Gamma_{\underline{r}}\gamma^{\hat{\imath}}$ in (2.54) to vanish, we obtain

$$F_{0i} = \partial_i\,(r\cos\theta)\,\,,\tag{2.55}$$

which can be interpreted as the BPS equation for a worldvolume BIon. It is straightforward to check that when (2.55) is satisfied the coefficients of $\gamma^{\hat{\imath}\hat{\jmath}}$ and $\gamma^{\hat{\imath}}\Gamma_{\underline{\theta}}$ vanish identically, so (2.48) is satisfied as a consequence of (2.55) provided that

$$U\,\sqrt{\det\left[r^2\bar{g}_{ij} + \partial_i r\partial_j r - F_{0i}F_{0j}\right]} = \Delta_5(r\sin\theta)'\,.\tag{2.56}$$

It can be verified that this is identically satisfied when F_{0i} is given by (2.55). We have thus shown that (2.55) is sufficient for preservation of 1/4 supersymmetry. We believe that it is also necessary; certainly, it is not difficult to check that there is no configuration preserving more than 1/4 supersymmetry.

We have now found the condition on r and F_{0i} required for preservation of supersymmetry. Inserting (2.56) into (2.12) and contracting with $\bar{g}^{ik}\partial_k r$ we get

$$\tilde{E}^i\partial_i r = \frac{\Delta_5}{(r\sin\theta)'}\,\bar{g}^{ij}\partial_i r\partial_j(r\cos\theta)\,,\tag{2.57}$$

which, when inserted back into (2.12), allows us to find the following relation between r and $\tilde{E}^i$:

$$(r\sin\theta)'\,\tilde{E}^i = \Delta_5\bar{g}^{ij}\left[\left(1 + \frac{\bar{g}^{kl}\partial_k r\partial_l r}{r^2}\right)\partial_j(r\cos\theta) - \partial_j r\,\frac{\bar{g}^{kl}\partial_k r\partial_l(r\cos\theta)}{r^2}\right].\tag{2.58}$$

JHEP11(1999)019

Then, using the identities

$$
\begin{aligned}
(r\sin\theta)' &= \left(1+\frac{\bar g^{kl}\partial_k r\partial_l r}{r^2}\right) r\cos\theta - \frac{\bar g^{kl}\partial_k r\partial_l (r\cos\theta)}{r}\,,\\
\bar g^{\hat\imath\hat\jmath}\partial_{\hat\imath} r\partial_{\hat\jmath} r\sin\theta &= -\frac{\bar g^{kl}\partial_k r\partial_l r}{r}\,(r\cos\theta)' + r'\,\frac{\bar g^{kl}\partial_k r\partial_l (r\cos\theta)}{r}\,,
\end{aligned}
\tag{2.59}
$$

one can check that (2.58) is equivalent to the two BPS equations (2.22) and (2.23). When combined with the Gauss law these are equivalent to the second order equation (2.24). We conclude that configurations satisfying this equation preserve 1/4 supersymmetry.

2.3 Multi-angle D5-brane solutions with 1/4 supersymmetry

We have just shown that any solution $r(\theta,\theta_{\hat\imath})$ to equation (2.24) preserves 1/4 supersymmetry. We earlier discussed the SO(5) invariant solution to this equation. We recall here that it has the form [8]

$$
r_\nu(\theta) = C\frac{(\eta(\theta))^{1/3}}{\sin\theta}\,, \tag{2.60}
$$

We shall now show that there exist many other, less-symmetric, solutions, in the near horizon region. Some may be found by considering fluctuations about the SO(5)-invariant solution. This analysis suggests that a large class of solutions may be found via an ansätze of the form

$$
r(\theta,\theta_{\hat\imath}) = r_\nu(\theta)\prod_{\hat\imath}\delta_{\hat\imath}(\theta_{\hat\imath})\,. \tag{2.61}
$$

When this is inserted into eq. (2.24) we find that

$$
\begin{aligned}
0 = {}& \sin^2\theta\sin^2\theta_3\sin\theta_2\left[3\sin^2\theta_4\cos\theta_4\psi_4+\sin^3\theta_4\psi_4'\right]+\\
&+\sin^2\theta\sin\theta_4\sin\theta_2\left[2\sin\theta_3\cos\theta_3\psi_3+\sin^2\theta_3\psi_3'\right]+\\
&+\sin^2\theta\sin\theta_4\left[\cos\theta_2\psi_2+\sin\theta_2\psi_2'\right]+\frac{\sin^2\theta\sin\theta_4}{\sin\theta_2}\psi_1'-\\
&-\sin^3\theta_4\sin^2\theta_3\sin\theta_2\frac{d}{d\theta}\left(\frac{r_\nu\sin^3\theta}{(r_\nu\sin\theta)'}\right)\times\\
&\times\left(\psi_4^2+\frac{\psi_3^2}{\sin^2\theta_4}+\frac{\psi_2^2}{\sin^2\theta_4\sin^2\theta_3}+\frac{\psi_1^2}{\sin^2\theta_4\sin^2\theta_3\sin^2\theta_2}\right),
\end{aligned}
\tag{2.62}
$$

where the new functions $\psi_{\hat\imath}(\theta_{\hat\imath})$, each depending on just one angular variable $\theta_{\hat\imath}$, are defined by

$$
\psi_{\hat\imath}(\theta_{\hat\imath}) \equiv \frac{\delta_{\hat\imath}'(\theta_{\hat\imath})}{\delta_{\hat\imath}(\theta_{\hat\imath})}\,, \tag{2.63}
$$

JHEP11(1999)019

the prime indicating differentiation with respect to the corresponding angular variable. The SO(5) invariant solution (2.60) satisfies the identity

$$\frac{d}{d\theta}\left(\frac{r_\nu(\theta)\sin^3\theta}{(r_\nu(\theta)\sin\theta)'}\right) = 3\sin^2\theta\,, \tag{2.64}$$

which can be used to simplify (2.62) to[2]

$$\begin{aligned} 0 = {} & \sin^2\theta_3\sin\theta_2\left[3\sin^2\theta_4\cos\theta_4\psi_4 + \sin^3\theta_4\psi_4' - 3\sin^3\theta_4\psi_4^2\right] + \\ & + \sin\theta_4\sin\theta_2\left[2\sin\theta_3\cos\theta_3\psi_3 + \sin^2\theta_3\psi_3' - 3\sin^2\theta_3\psi_3^2\right] + \\ & + \sin\theta_4\left[\cos\theta_2\psi_2 + \sin\theta_2\psi_2' - 3\sin\theta_2\psi_2^2\right] + \\ & + \frac{\sin\theta_4}{\sin\theta_2}\left[\psi_1' - 3\psi_1^2\right] \end{aligned} \tag{2.65}$$

A simple way to find solutions of this equation is to require $\psi_{\hat{\imath}}$ to satisfy the Bernoulli equations[3]

$$\psi_4' + 3\cot\theta_4\,\psi_4 = 3\psi_4^2\,, \tag{2.66}$$

$$\psi_3' + 2\cot\theta_3\,\psi_3 = 3\psi_3^2\,, \tag{2.67}$$

$$\psi_2' + \cot\theta_2\,\psi_2 = 3\psi_2^2\,, \tag{2.68}$$

$$\psi_1' = 3\psi_1^2\,. \tag{2.69}$$

These equations admit the trivial solutions $\psi_{\hat{\imath}}(\theta_{\hat{\imath}}) = 0$, whereby we recover the previous SO(5) invariant solution, but they are also solved by

$$\frac{1}{\psi_4} = c_4\sin^3\theta_4 + \frac{3}{2}\sin\theta_4\cos\theta_4 - \frac{3}{2}\sin^3\theta_4\log\tan\frac{\theta_4}{2}\,, \tag{2.70}$$

$$\frac{1}{\psi_3} = c_3\sin^2\theta_3 + 3\sin\theta_3\cos\theta_3\,, \tag{2.71}$$

$$\frac{1}{\psi_2} = c_2\sin\theta_2 - 3\sin\theta_2\log\tan\frac{\theta_2}{2}\,, \tag{2.72}$$

$$\frac{1}{\psi_1} = c_1 - 3\theta_1\,, \tag{2.73}$$

where $c_{\hat{\imath}}$ are constants of integration.

Due to the functional form of our ansatz (2.61), the new solutions cannot represent isolated singularities. In fact, the set of singularities in the radial function

[2] In fact, by requiring $\frac{d}{d\theta}\left(\frac{r_{q,\nu}(\theta)\sin^3\theta}{(r_{q,\nu}(\theta)\sin\theta)'}\right) = (3+q)\sin^2\theta$, we can obtain more general ansätze compatible with the separation of variables whose dependence on the polar angle $r_{q,\nu}(\theta)$ is different from the one in the SO(5) invariant solution (2.60). We will not consider these solutions here.

[3] This is certainly not the most general solution to eq. (2.65). One may introduce three separation constants following the usual separation of variables prescription.

JHEP11(1999)019

$r(\theta, \theta_{\hat{\imath}})$ is the union of the corresponding sets of singularities of the different product terms appearing in (2.61). Let us analyse the singularity structure of the new functions $\delta_{\hat{\imath}}(\theta_{\hat{\imath}})$. They are formally given by

$$\delta_{\hat{\imath}}(\theta_{\hat{\imath}}) = a_{\hat{\imath}} e^{\int \psi_{\hat{\imath}}(\theta_{\hat{\imath}})\, d\theta_{\hat{\imath}}}\,, \tag{2.74}$$

which can not be solved analytically, in general. It is nevertheless possible to study their behaviour near singular points. Given a Bernoulli equation of the type :

$$\psi' + q\, \cot\theta\, \psi = 3\, \psi^2\,, \qquad q = 1, 2, 3 \tag{2.75}$$

it can be checked that,

$$\psi \sim \begin{cases} \dfrac{q-1}{3} \dfrac{1}{\theta} & \text{if } q \neq 1 \\ -\dfrac{1}{3} \dfrac{1}{\theta \log\theta} & \text{if } q = 1 \end{cases} \qquad \theta \sim 0\,, \tag{2.76}$$

$$\psi \sim \begin{cases} \dfrac{q-1}{3} \dfrac{1}{\theta - \pi} & \text{if } q \neq 1 \\ -\dfrac{1}{3} \dfrac{1}{(\theta-\pi)\log(\pi-\theta)} & \text{if } q = 1 \end{cases} \qquad \theta \sim \pi\,, \tag{2.77}$$

from which one can conclude that new functions δ are non-singular around $\theta \sim 0, \pi$. Since the function $1/\psi$ changes sign when θ runs from 0 to π, we conclude that it should vanish for some value $\theta = \alpha$ defined by

$$\frac{1}{\psi(\alpha)} = 0\,, \qquad \alpha \neq 0, \pi\,. \tag{2.78}$$

Since (2.75) can be rewritten for $\psi \neq 0$ as

$$-\frac{d}{d\theta}\left(\frac{1}{\psi}\right) + q \cot\theta\, \frac{1}{\psi} = 3\,, \tag{2.79}$$

it is straightforward to derive the behaviour

$$\psi \sim -\frac{1}{3}\frac{1}{\theta - \alpha}\,, \qquad \theta \sim \alpha\,, \tag{2.80}$$

which is independent of q. Thus all non-polar angles have branch points of the type

$$\delta \sim \frac{1}{(\theta - \alpha)^{1/3}}\,. \tag{2.81}$$

The analysis for $q = 0$, which applies to the angle θ_1, is straightforward. In this case $\delta_1(\theta_1)$ can be computed analytically:

$$\delta_1(\theta_1) = \frac{a_1}{(\alpha_1 - \theta_1)^{1/3}}\,. \tag{2.82}$$

JHEP11(1999)019

Note that the reality of $r(\theta, \theta_{\hat{\imath}})$ restricts θ_1 to the interval $(0, \alpha_1)$, so we have a case of partial wrapping.[4] That is, the branch cut at $\theta_1 = \alpha_1$ represents a locus of points at which the D5-brane leaves the 5-sphere. These configurations are therefore not 'baryonic branes' as defined in the introduction, although they are 1/4 supersymmetric configurations of the D5-brane.

Inserting the ansatz (2.61) into equations (2.22)–(2.23), we can compute the conjugate momentum E^i for any solution of the system (2.66)–(2.69):

$$
\begin{aligned}
E^4 &= R^4 \sin^2\theta \sqrt{\det{}^4 g}\, \psi_4(\theta_4) && (2.83)\\
E^3 &= R^4 \sin^2\theta \sqrt{\det{}^4 g}\, \frac{\psi_3(\theta_3)}{\sin^2\theta_4} && (2.84)\\
E^2 &= R^4 \sin^2\theta \sqrt{\det{}^4 g}\, \frac{\psi_2(\theta_2)}{\sin^2\theta_4 \sin^2\theta_3} && (2.85)\\
E^1 &= R^4 \sin^2\theta \sqrt{\det{}^4 g}\, \frac{\psi_1(\theta_1)}{\sin^2\theta_4 \sin^2\theta_3 \sin^2\theta_2} && (2.86)\\
E^\theta &= E_\nu(\theta) \sqrt{\det{}^4 g} - \frac{3}{2} R^4 \eta(\theta) \sqrt{\det{}^4 g}\, \times \\
&\quad \times \left[(\psi_4(\theta_4))^2 + \frac{(\psi_3(\theta_3))^2}{\sin^2\theta_4} + \frac{(\psi_2(\theta_2))^2}{\sin^2\theta_4 \sin^2\theta_3} + \frac{(\psi_1(\theta_1))^2}{\sin^2\theta_4 \sin^2\theta_3 \sin^2\theta_2} \right]. && (2.87)
\end{aligned}
$$

A simple calculation suffices to show that the E^i satisfy the Gauss law constraint (2.8) as a consequence of the differential equations satisfied by the functions $\psi_{\hat{\imath}}$. To compute the energy of the new configurations, we should compute their topological charge. Using the ansatz (2.61) in (2.19), this can be written as

$$
\begin{aligned}
\mathcal{Z}_5 &= \sqrt{\det{}^4 g} \prod_{\hat{\imath}} \delta_{\hat{\imath}}(\theta_{\hat{\imath}}) \left(\mathcal{Z}(\theta) + \frac{3}{2} r_0(\theta) \frac{\eta(\theta)}{\sin\theta} R^4 \times \right. && (2.88)\\
&\quad \left. \times \left[(\psi_4(\theta_4))^2 + \frac{(\psi_3(\theta_3))^2}{\sin^2\theta_4} + \frac{(\psi_2(\theta_2))^2}{\sin^2\theta_4 \sin^2\theta_3} + \frac{(\psi_1(\theta_1))^2}{\sin^2\theta_4 \sin^2\theta_3 \sin^2\theta_2} \right] \right)
\end{aligned}
$$

The terms associated with new ($\psi_{\hat{\imath}}$-dependent) contributions are such that the new solutions have a positively defined topological charge whenever the 'old' (SO(5)-invariant) contribution is non-negative. This is the case, for the full range of θ, when $\nu = 0$.As expected from our general discussion, $\mathcal{Z}_5$ can be rewritten as a divergence, $\mathcal{Z}_5 = \partial_i \mathcal{Z}_5^i$, at least in the near-horizon ($a = 0$) limit. By inserting the conjugate momenta (2.83-2.87) into (2.20), taking the near horizon limit and using the ansätze (2.61) for our new radial functions, we find the components $\vec{\mathcal{Z}}_5$ to be

$$
\begin{aligned}
\mathcal{Z}_5^\theta &= E^\theta\, r \cos\theta + R^4 \sin^5\theta \sqrt{det\, {}^4 g}\, r \\
\mathcal{Z}_5^{\hat{\imath}} &= E^{\hat{\imath}}\, r \cos\theta\, . && (2.89)
\end{aligned}
$$

[4] We may assume that $\alpha \leq 2\pi$ since we may otherwise shift θ_1 by 2π.

JHEP11(1999)019

To summarize, we have found many new 1/4 supersymmetric configurations of a D5-brane in the near-horizon geometry of a D3-brane background. None of these new solutions has a straightforward interpretation as a 'baryonic brane' because (i) they have branch cut singularities rather than poles, and (ii) they only partially wrap the 5-sphere. However, it seems clear that there must be a large class of other 1/4 supersymmetric solutions that are not captured by our ansatz. We have argued earlier that these are likely to include solutions with multiple isolated point singularities, but it appears to be difficult to find explicit solutions of this type.

3. Baryonic M5-brane

We start from the M5-brane solution of $D = 11$ supergravity. The metric and 4-form field strength $F_{(4)} = dC^{(3)}$ are

$$\begin{aligned} ds^2_{(11)} &= U^{-1/3}\left[ds^2(\mathbb{E}^{(1,4)}) + dX_5^2\right] + U^{2/3}\left[dr^2 + r^2\, d\Omega^2_{(4)}\right], \\ F_{(4)} &= -3\, R^3 \omega_4\,, \end{aligned} \tag{3.1}$$

where $d\Omega_4^2$ is the SO(5) invariant metric on the unit 4-sphere and ω_4 is its volume 4-form. The function U is

$$U = a + \left(\frac{R}{r}\right)^3, \qquad R^3 = \pi\, N l_p^3\,. \tag{3.2}$$

We will put a probe M5-brane in this background and look for solutions for which the M5-brane wraps around the S^4 factor. We shall start from the Lorentz covariant M5-brane action [19]; a useful review is [20] to which we refer for details of this formalism and references to related work. We use the conventions of [21], with minor modifications. The action is constructed from the induced worldvolume metric h and a 2-form worldvolume potential A with 'modified' 3-form field strength

$$H = dA - C^{(3)}\,. \tag{3.3}$$

Here, $C^{(3)}$ should be understood as the pullback of the 3-form potential of the background; we will use the same symbol for a spacetime form and its pullback since it will be clear which is meant from the context. Thus, the Bianchi identity for H can be written as

$$dH = -F_{(4)}\,. \tag{3.4}$$

There is an additional field in the Lorentz covariant action, the 'PST scalar', but it can be removed by a gauge transformation at the cost of breaking manifest Lorentz covariance.

Let $\xi^\mu = (t, \sigma^I)$ be the M5-brane worldvolume coordinates, and define the *worldspace* tensor

$$\tilde{H}^{IJ} = \frac{1}{6\sqrt{\det g}}\varepsilon^{IJKLMN} H_{KLM}\,, \tag{3.5}$$

JHEP11(1999)019

where g is the induced *worldspace* metric. Let

$$\tilde{H}_{\mu\nu} = h_{\mu I} h_{\nu J} \tilde{H}^{IJ} \,. \tag{3.6}$$

This is not a worldvolume tensor but it is derived from a tensor by choice of the temporal gauge for the PST field. In the same gauge, and for unit M5-brane tension, the M5-brane action is

$$S = -\int d^6\xi \left\{ \sqrt{-\det(h+\tilde{H})} + V^P h_{P0} \right\} + \frac{1}{4}\int \left[C^{(6)} + 2H\wedge C^{(3)} + H\wedge H \right], \tag{3.7}$$

where

$$V^P = \frac{1}{24} g^{PN} \varepsilon^{IJKLM} H_{KLM} H_{IJN} \tag{3.8}$$

and $C^{(6)}$ is the (pullback of) the 6-form potential dual to $C^{(3)}$ (defined in backgrounds that satisfy the $D = 11$ supergravity field equations).

Notice that in the given background, the Bianchi identity (3.4) becomes

$$dH = 3\, R^3 \omega_4 \,. \tag{3.9}$$

As in the D5-brane case we shall find that this will require infinite radial deformations carrying M2-brane charge, analogous to the self-dual string solitons on the M5-brane [22]. Pursuing this analogy, we may think of the entire configuration as one involving three branes, background, probe and 'soliton', intersecting according to the array on the right.

M5:	1 2 3 4 5 _ _ _ _ _ _	background
M5:	_ _ _ _ 5 6 7 8 9 _	probe
M2:	_ _ _ _ 5 _ _ _ _ ♮	soliton.

Configuration 2: An M2-brane intersecting an M5-brane probe in an M5-brane background. The 5-direction is the direction of the string-like intersection. The ♮-direction is radially transverse to both this intersection and the M5-brane probe.

The probe M5-brane has bosonic worldvolume fields X^a ($a = 1, \ldots, 4$), X^0, X^5, r and Θ^i ($i = 1, \ldots, 4$) where Θ^i are now four angles parametrising S^4, and a three-form field strength H. Let $\sigma^I = (\sigma, \theta^i)$ be the worldspace coordinates with θ^i parameterising the 4-sphere. We shall choose the static gauge

$$X^0 = t\,, \qquad X^5 = \sigma\,, \qquad \Theta^i = \theta^i\,, \qquad i = 1,2,3,4\,, \tag{3.10}$$

appropriate to an M5-brane wrapped on a 4-sphere at a (variable) radius in the 5-space transverse to the background M5-brane. We also set

$$X^1 = X^2 = X^3 = X^4 = 0\,. \tag{3.11}$$

In other words, we assume that r is the only 'active' scalar. We also assume that $\partial_5 r = 0$, so that r is a function on the 4-sphere, with isolated singularities. For static

JHEP11(1999)019

configurations we now have

$$h_{\mu\nu} = \begin{pmatrix} -U^{-1/3} & 0 & 0 \\ 0 & U^{-1/3} & 0 \\ 0 & 0 & g_{ij} \end{pmatrix}, \tag{3.12}$$

where

$$g_{ij} = U^{2/3}\left(r^2 \bar{g}_{ij} + \partial_i r \partial_j r\right). \tag{3.13}$$

The metric $\bar{g}$ is now the SO(4)-invariant metric on the unit 3-sphere.

For the background we are considering, $C^{(3)}_{5ij}$ vanishes, so we may assume that

$$H_{5ij} = 0\,. \tag{3.14}$$

This means that the worldspace vector density V^I vanishes and that the only non-vanishing components of $\tilde{H}^{IJ}$ are

$$\tilde{H}^{5i} = \frac{U^{1/6}}{\sqrt{\det g}}\,\Pi^i\,, \tag{3.15}$$

where

$$\Pi^i \equiv \frac{1}{6}\varepsilon^{ijkl} H_{jkl}\,. \tag{3.16}$$

In terms of this new variable the Bianchi identity (3.9) is

$$\partial_i \Pi^i = 3\,R^3\,\sqrt{\det \bar{g}}\,. \tag{3.17}$$

Since $g_{5i} = 0$, the only non-vanishing components of $\tilde{H}_{\mu\nu}$ are

$$\tilde{H}_{5i} = g_{55} g_{ij} \tilde{H}^{5j} = \frac{U^{-1/6}}{\sqrt{\det g}}\, g_{ij}\Pi^j\,. \tag{3.18}$$

3.1 Energy bound

We shall now derive an energy bound for baryonic M5-branes. It follows from the hamiltonian formulation of the super M5-brane in a general background [21], that the energy density $\mathcal{H}$ of a static configuration, in the background we consider and subject to the restrictions discussed above, is such that

$$\mathcal{H}^2 = U^{-2/3}\left[\det g + \Pi^i \Pi^j g_{ij}\right], \tag{3.19}$$

where g_{ij} is given by (3.13). This expression is invariant under the SO(5) isometry group of the 4-sphere, but minimum energy configurations can be at most SO(4) invariant. We shall later wish to exploit this possibility so we write the 4-sphere metric as

$$ds^2 = d\theta^2 + \sin^2\theta\, d\Omega_3^2\,, \tag{3.20}$$

JHEP11(1999)019

where $d\Omega_3^2$ is the SO(4)-invariant metric on the unit 3-sphere. Let $\theta^{\hat{\imath}}$ ($\hat{\imath} = 1, 2, 3$) parametrize the 3-sphere, so that $\theta^i = (\theta, \theta^{\hat{\imath}})$. We may now rewrite (3.19) as

$$\begin{aligned} \mathcal{H}^2 &= \Delta_4^2 \left(r^2 + r'^2 + \bar{g}^{\hat{\imath}\hat{\jmath}} \partial_{\hat{\imath}} r \partial_{\hat{\jmath}} r\right) + \left(\Pi^\theta r'\right)^2 + \left(\Pi^{\hat{\imath}} \partial_{\hat{\imath}} r\right)^2 + \\ &\quad + 2(\Pi^\theta r')(\Pi^{\hat{\imath}} \partial_{\hat{\imath}} r) + \left(\Pi^\theta r\right)^2 + r^2 \Pi^{\hat{\imath}} \Pi^{\hat{\jmath}} \bar{g}_{\hat{\imath}\hat{\jmath}}\,, \end{aligned} \tag{3.21}$$

where

$$\Delta_4 \equiv U\, r^3 \sqrt{\det \bar{g}}\,. \tag{3.22}$$

This can be further rewritten as

$$\mathcal{H}^2 = \mathcal{Z}_4^2 + \left[\Delta_4 \left(r\cos\theta\right)' + \Pi^i \partial_i \left(r \sin\theta\right)\right]^2 + |\Delta_4 \bar{g}^{\hat{\imath}\hat{\jmath}} \partial_{\hat{\jmath}} r + r\, \Pi^{\hat{\imath}}|^2\,, \tag{3.23}$$

where

$$\mathcal{Z}_4 = \Delta_4 \left(r \sin\theta\right)' - \Pi^i \partial_i \left(r\cos\theta\right) \tag{3.24}$$

and $||^2$ indicates contraction with the $\hat{\imath}\hat{\jmath}$ components of the four sphere metric $\bar{g}_{ij}$, and $\bar{g}^{\hat{\imath}\hat{\jmath}}$ are the $\hat{\imath}\hat{\jmath}$ components of the inverse 4-sphere metric $\bar{g}^{ij}$.

Using the Bianchi identity (3.9), one can show that $\mathcal{Z}_4 = \partial_i \mathcal{Z}_4^i$ where

$$\begin{aligned} \mathcal{Z}_4^\theta &= -\Pi^\theta\, r\cos\theta + \sqrt{\det \bar{g}} \sin\theta \left(a\, \frac{r^4}{4} + r\, R^3\right), \\ \mathcal{Z}_4^{\hat{\imath}} &= -\Pi^{\hat{\imath}}\, r\cos\theta\,. \end{aligned} \tag{3.25}$$

From (3.23), and the fact that $\mathcal{Z}_4$ is a divergence, we deduce the bound

$$\mathcal{H} \geq |\mathcal{Z}_4| \tag{3.26}$$

with equality when

$$\Pi^{\hat{\imath}} = -\Delta_4 \frac{\bar{g}^{\hat{\imath}\hat{\jmath}} \partial_{\hat{\jmath}} r}{r}\,, \tag{3.27}$$

$$\Pi^\theta = -\frac{\Delta_4}{(r\sin\theta)'} \left((r\cos\theta)' - \bar{g}^{\hat{\imath}\hat{\jmath}} \partial_{\hat{\imath}} r \partial_{\hat{\jmath}} r \frac{\sin\theta}{r}\right). \tag{3.28}$$

Because $\mathcal{H}$ is, by assumption, independent of the worldspace coordinate σ we may interpret its integral over the remaining worldspace coordinates, parameterising S^4, as the tension T of a string with string coordinate σ. It then follows from the bound (3.26) that

$$T \geq \int d^4\theta\, |\mathcal{Z}_4| \geq \left| \int d^4\theta \mathcal{Z}_4 \right|. \tag{3.29}$$

The first inequality is saturated by solutions of (3.30). The second inequality will be saturated too provided that $\mathcal{Z}_4$ does not change sign in the integration region. If this is the case then the tension T can be expressed as a surface integral over a sum of N 3-spheres surrounding the N singular points on the 4-sphere. Each such

JHEP11(1999)019

surface term can be considered to be the energy/charge of an M2-brane attached to the 4-sphere at that point.

By combining the BPS conditions (3.27) and (3.28) with the Bianchi identity (3.17) we deduce that the energy density is minimised when the radial function r is a solution of the second order equation

$$\left[\frac{\Delta_4}{(r\sin\theta)'}\left((r\cos\theta)' - \frac{\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\imath}} r\,\partial_{\hat{\jmath}}(r\sin\theta)}{r}\right)\right]' + \partial_{\hat{\imath}}\left[\Delta_4 \frac{\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\jmath}} r}{r}\right] = -3\,R^3\sqrt{\det\bar{g}}\,. \quad (3.30)$$

In the remainder of this subsection we shall restrict ourselves to the near-horizon $a=0$ geometry. To simplify the task of finding explicit solutions of equations (3.27) and (3.28) in this case, we will restrict our attention to an SO(4)-invariant ansatze for which the radial function depends only on θ. In this case only the Π^θ component of Π^i is non-zero, and

$$\Pi^\theta = \sqrt{\det\bar{g}^{(3)}}\,\Pi(\theta)\,, \quad (3.31)$$

where $\Pi(\theta)$ is a function only of θ, and $\bar{g}^{(3)}$ is the SO(4)-invariant metric on the unit 3-sphere. The Bianchi identity (3.9) now reduces to

$$\Pi' = 3\,R^3\sin^3\theta\,. \quad (3.32)$$

This has the solution

$$\Pi(\theta) = -R^3\left[\cos\theta\left(\sin^2\theta + 2\right) + 2(2\nu - 1)\right], \quad (3.33)$$

where ν is an integration constant which, as in the D5-brane case, can be restricted to lie in the interval $[0,1]$.

It will be convenient to set

$$\Delta_4 = \sqrt{\det\bar{g}^{(3)}}\,\Delta(\theta)\,, \quad (3.34)$$

since $\Delta = R^3\sin^3\theta$ is a function only of θ. For SO(4)-invariant M5-brane configurations the first BPS equation (3.27) is trivially satisfied while (3.28) reduces to

$$\frac{r'}{r} = \frac{\Delta\sin\theta - \Pi\cos\theta}{\Delta\cos\theta + \Pi\sin\theta}\,. \quad (3.35)$$

We may also set

$$\mathcal{Z}_4 = \sqrt{\det\bar{g}^{(3)}}\,\mathcal{Z}\,, \quad (3.36)$$

where $\mathcal{Z}$ is now a function only of θ. Using (3.35) we find that

$$\mathcal{Z} = r\,\frac{(\Delta\sin\theta - \Pi\cos\theta)^2 + (\Delta\cos\theta + \Pi\sin\theta)^2}{(\Delta\cos\theta + \Pi\sin\theta)}\,. \quad (3.37)$$

Thus $\mathcal{Z}$ has a definite sign provided that the denominator does not vanish. As

$$\Delta\cos\theta + \Pi\sin\theta = -2R^3\sin\theta\left[\cos\theta - 1 + 2\nu\right], \quad (3.38)$$

JHEP11(1999)019

the denominator of (3.37) has a definite sign for $\theta \in [0, \pi]$ if $\nu = 0$. In this case the solution of the first-order differential equation (3.35) can be easily shown to be:

$$r = r_0 \sec \frac{\theta}{2}\,, \tag{3.39}$$

where r_0 is the value of $r(\theta)$ at $\theta = 0$. This solution was previously found in [13] in the context of a D4-brane probe in the D4-brane background geometry. It is straightforward to verify that the function given in (3.39) solves the second-order ODE obtained from (3.30) by setting $a = 0$ and restricting r to depend only on θ, namely:

$$\left[\sin^3\theta\,\frac{r'\cos\theta - r\sin\theta}{r'\sin\theta + r\cos\theta}\right]' = -3\sin^3\theta\,. \tag{3.40}$$

The function (3.39) represents an M5-brane wrapped on the S^4 with a spike at the point $\theta = \pi$, where $r(\theta)$ diverges. This spike extends along the axis $\theta = \pi$. From our previous results it is a simple exercise to compute the value of the tension T for the configuration represented by (3.39). Indeed, if we recall that $\mathcal{Z}_4$ is a total derivative, the integral of $\mathcal{H}$ over the four-sphere can be immediately evaluated. The energy per unit surface of the spike obtained from this calculation is:

$$T_{M5}\, V_{(3)} |\Pi\,(\pi)|\,, \tag{3.41}$$

where $V_{(3)} = 2\pi^2$ is the volume of the unit three sphere, T_{M5} is the tension of the M5-brane which, up to now, was considered to be one and which is given by:

$$T_{M5} = \frac{1}{(2\pi)^5\, l_p^6}\,, \tag{3.42}$$

and $\Pi(\pi)$ can be extracted from (3.33) with $\nu = 0$. An elementary calculation shows that

$$T_{M5}\, V_{(3)}\, |\Pi(\pi)| = N\, T_{M2}\,, \tag{3.43}$$

where

$$T_{M2} = \frac{1}{(2\pi)^2\, l_p^3} \tag{3.44}$$

is the tension of the M2-brane. This clearly shows that the spike at $\theta = \pi$ can be interpreted as a bundle of N M2-branes emanating from the M5-brane. The relation of our configuration of M5-branes and M2-branes with a configuration of D4-branes and IIA fundamental strings is easily understood if we reduce table 2 along the 5-direction.

As in the D5-brane case, we could have considered configurations with $\nu \neq 0$. The corresponding solutions of the BPS equation can be extracted from those found in [13] for the D4-brane in the near-horizon region of the D4-brane geometry. These configurations of the M5-brane do not completely wrap the S^4 and they extend into the M5-brane 'throat' region all the way to $r = 0$, in complete analogy with what happens in the D5-brane case. Similarly, one can evaluate the energy per unit surface of the spikes of the $\nu \neq 0$ solutions and show that their energy per unit surface corresponds to a bundle of k M2-branes with $k < N$.

JHEP11(1999)019

3.2 Full M5 background

We are now going to study the embedding of the probe M5-brane in the full M5-brane metric, which corresponds to taking $a = 1$ in the harmonic function U of equation (3.2). We shall restrict ourselves to the analysis of the SO(4)-invariant embeddings. In this case, as was shown in [8], it is more convenient to work in a new set of variables (z, ρ), related to (r, θ) as follows:

$$z = -r\cos\theta\,, \qquad \rho = r\sin\theta\,. \tag{3.45}$$

It is clear from (3.45) that z can take values in the interval $(-\infty, +\infty)$ whereas $0 \le \rho < \infty$. In these new coordinates the M5-brane embedding is determined by a function $z(\rho)$. By using the relation between the (z, ρ) and (r, θ) coordinates, it is not difficult to relate $z(\rho)$ and its derivative to $r(\theta)$ and $r'(\theta) = dr/d\theta$. After a short calculation one gets:

$$\frac{dz}{d\rho} = \frac{r\sin\theta - \cos\theta\, r'}{r\cos\theta + \sin\theta\, r'}\,. \tag{3.46}$$

By using (3.46) one can convert the BPS condition (3.28) into a first-order differential equation for $z(\rho)$. To this end, we introduce the function

$$\hat{\Delta}(\rho, z) = \rho^3\left(1 + \frac{R^3}{[\rho^2 + z^2]^{3/2}}\right) \tag{3.47}$$

It is easy to show that (3.28) is now equivalent to

$$\frac{dz}{d\rho} = \frac{\Pi\Big(\arctan\left(-\rho/z\right)\Big)}{\hat{\Delta}(\rho, z)}\,, \tag{3.48}$$

where Π is the same function as in equation (3.33) and its argument in (3.48) is obtained after inverting the relation (3.45) between both coordinate systems.

The condition (3.48) can also be obtained by studying the energy of the M5-brane in the new variables. Let us define, as in equation (3.29), the tension T as the energy density integrated over the four worldspace coordinates θ^i $(i = 1, \ldots, 4)$. It is not difficult to change variables in this integral and express the result as the following integral over ρ:

$$T = V_{(3)}\int d\rho\sqrt{\left(\Pi\,\frac{dz}{d\rho} + \hat{\Delta}\right)^2 + \left(\hat{\Delta}\,\frac{dz}{d\rho} - \Pi\right)^2} \tag{3.49}$$

In this equation, and in what follows, Π depends on ρ and $z(\rho)$ as in (3.48). The right-hand side of equation (3.49) contains a remarkably simple sum of squares. This simplicity is a reflection of how convenient are the new coordinates (z, ρ) for

JHEP11(1999)019

this $a = 1$ case, as will be confirmed below. Notice that the second term in (3.49) vanishes precisely when the BPS condition (3.48) holds. Clearly, if we define:

$$X \equiv \Pi \frac{dz}{d\rho} + \hat{\Delta} \tag{3.50}$$

we obtain the following bounds:

$$T \geq V_{(3)} \int d\rho \, |X| \geq V_{(3)} \Big| \int d\rho \, X \Big| . \tag{3.51}$$

Obviously, the first bound in (3.51) is saturated when the BPS condition (3.48) holds. We will show below that the second inequality is also saturated if equation (3.48) is satisfied. Moreover, it is easy to prove that X is a total derivative with respect to ρ, namely:

$$X = \frac{d}{d\rho} \left[z\Pi + \left(\frac{1}{4} + \frac{R^3}{[\rho^2 + z^2]^{3/2}} \right) \rho^4 \right] . \tag{3.52}$$

It is important to stress the fact that equation (3.52) follows from the explicit form of Π given in (3.33) and, thus, is valid for any function $z(\rho)$. Actually, the derivative of Π with respect to ρ for an arbitrary function $z(\rho)$ is given by:

$$\frac{d\Pi}{d\rho} = \frac{3R^3\rho^3}{[\rho^2 + z^2]^{5/2}} \left(\rho \frac{dz}{d\rho} - z \right) . \tag{3.53}$$

Using (3.53) to evaluate the right-hand side of (3.52), one easily verifies that the result is the function defined in equation (3.50). Moreover, the value of X when the BPS condition is satisfied is given by:

$$X_{BPS} = \left[\frac{dz}{d\rho} + \left(\frac{dz}{d\rho} \right)^{-1} \right] \Pi \, . \tag{3.54}$$

As $\hat{\Delta} \geq 0$, the sign of $dz/d\rho$ for any $z(\rho)$ satisfying (3.48) is the same as the sign of Π. Therefore, it follows that

$$X_{BPS} \geq 0 \, , \tag{3.55}$$

which implies, as was previously claimed, that the second inequality in (3.51) is saturated if the first one is saturated.

The BPS equation (3.48) is the same as the one appearing in the study of a D4-brane probe in the full D4-brane geometry. Its analytical solution was found in [14]. It turns out that $z(\rho)$ can be represented implicitly by the equation

$$z = z_\infty + \frac{R^3}{\rho^2} \left[\frac{z}{\sqrt{\rho^2 + z^2}} + 1 - 2\nu \right] , \tag{3.56}$$

where z_∞ is a constant of integration. It can be verified that this function satisfies the second-order differential equation (3.30), as required to minimise the energy.

JHEP11(1999)019

The behaviour of the solution (3.56) has been analyzed in detail in [14]. It was found there that, for regions close to the horizon, the embedding represented by the function $z(\rho)$ coincides with the solutions found for $a = 0$. However, for the asymptotic region $\rho \to \infty$, the behaviour of both types of solutions is completely different. Indeed, it follows from (3.47) and (3.48) that for $\rho \to \infty$ and z fixed (which corresponds to $\theta \to \pi/2$), $dz/d\rho$ decreases as ρ^{-3}, which implies that $z \to$constant as $\rho \to \infty$, i.e. the asymptotic shape of the brane is just a plane. This constant asymptotic value of z is precisely the integration constant z_∞, as can be verified directly from (3.56). Notice that this is very natural from the physical point of view since our metric is asymptotically flat and one expects the minimal energy configuration of a tensile object in a flat metric to be such that the object is not bent at all.

Under certain conditions our solution (3.56) contains a tubular region which has one of its ends near the horizon and can locally be described by the equation $\rho =$constant. Since $|dz/d\rho|$ is then large, these portions of the brane are not located in the asymptotic region $\rho \to \infty$ in which, as we argued above, $dz/d\rho \to 0$. These tubes therefore spread out and approach the plane $z = z_\infty$ as one moves away from the near-horizon region. Detailed study [14] shows that for large $|z_\infty|$ the transition between these two regimes is very fast. Notice that these spikes, which have infinite length in the near-horizon approximation, have actually a finite length (of the order of $|z_\infty|$) in the full geometry. In this sense one can say that the full metric regularizes the singularities of the near-horizon description. Moreover, it can be verified that the energy of the tubes per unit surface equals that expected for a bundle of M2-branes. From this analysis we get a picture of the HW effect in M-theory in which M5-branes are connected by M2-branes which, from the worldvolume point of view, can be regarded as wormholes connecting the test M5 brane to the horizon of the metric created by the background M5-branes.

JHEP11(1999)019

3.3 Supersymmetry

We will now show that configurations saturating the bound (3.26) preserve 1/4 of the bulk supersymmetry. Our starting point is again (1.1). The explicit form of Γ_κ for the M5-brane can be found in [20]. Omitting some terms which are manifestly zero for the configurations we consider, and passing to the temporal gauge for the PST scalar, we have

$$\sqrt{-\det(h+\tilde{H})}\,\Gamma_\kappa = \frac{1}{2}\sqrt{-\det h}\,\Gamma^{\underline{0}}\Gamma_{IJ}\tilde{H}^{IJ} - \frac{1}{5!}U^{-1/6}\Gamma^{\underline{0}}\epsilon^{I_1\ldots I_5}\Gamma_{I_1\ldots I_5}\,. \qquad (3.57)$$

Again $\Gamma_{IJ\ldots}$ are antisymmetrized products of the (reducible) worldvolume gamma matrices $\Gamma_I = \partial_I X^m E_m{}^{\underline{m}}\Gamma_{\underline{m}}$, where $\Gamma_{\underline{m}}$ are the constant $D = 11$ Dirac matrices and $E_m{}^{\underline{m}}$ is the spacetime elfbein. In the static gauge and for static configurations with

$X^1 = X^2 = X^3 = X^4 = 0$ we have

$$\begin{aligned} \Gamma_0 &= U^{-1/3}\Gamma_{\underline{0}}\,, \\ \Gamma_i &= U^{1/3}\, r\gamma_i + U^{1/3}\partial_i r\Gamma_{\underline{\natural}}\,. \end{aligned} \tag{3.58}$$

In this expression we have introduced the matrix

$$\gamma_i = e_i{}^{\underline{i}}\Gamma_{\underline{i}}\,, \tag{3.59}$$

where $e_i{}^{\underline{i}}$ is the S^4 vierbein. Thus

$$\{\gamma_i, \gamma_j\} = 2\bar{g}_{ij}\,. \tag{3.60}$$

For the M5-brane background, the Killing spinors take the form

$$\chi = U^{-1/12}\epsilon\,, \tag{3.61}$$

where ϵ is covariantly constant on $\mathbb{E}^{(1,5)} \times \mathbb{E}^5$ subject to a 1/2 supersymmetry breaking condition imposed by the background, which we discuss below. The condition for preservation of supersymmetry (1.1) is thus reduced to

$$\Gamma_\kappa \epsilon = \epsilon\,. \tag{3.62}$$

In polar coordinates for $\mathbb{E}^5$, with $\Gamma_{\underline{\hat{4}}} \equiv \Gamma_{\underline{\theta}}$, we have

$$\epsilon = e^{\frac{\theta}{2}\Gamma_{\underline{\natural\theta}}} \prod_{\hat{j}=1}^{3} e^{-\frac{\theta^{\hat{j}}}{2}\Gamma_{\underline{\hat{j}\hat{j}+1}}}\,\epsilon_0\,, \tag{3.63}$$

where ϵ_0 is a constant spinor satisfying

$$\Gamma_{\underline{6789\natural}}\epsilon_0 = \epsilon_0\,. \tag{3.64}$$

There are again additional Killing spinors in the near-horizon limit ($a = 0$) but these play no role in our analysis for the same reason as in the D5-brane case.

The supersymmetry condition (3.62) gives

$$\begin{aligned} U^{-1/3}\sqrt{\det\left(g_{ij} + K_i K_j\right)}\epsilon = \Big[&\Delta_4 r\Gamma_{\underline{0}}\gamma_* - \Pi^i\partial_i r\Gamma_{\underline{05\natural}} - \\ &-\gamma_i\Gamma_{\underline{\natural}}\left(\Pi^i r\Gamma_{\underline{05\natural}} + U\, r^3\sqrt{\det\bar{g}}\,\bar{g}^{ij}\partial_j r\Gamma_{\underline{0}}\gamma_*\right)\Big]\,\epsilon\,, \end{aligned} \tag{3.65}$$

where

$$K_i = \frac{1}{\sqrt{\det g}}\, g_{ij}\Pi^j \tag{3.66}$$

and

$$\gamma_* = \Gamma_{\underline{56789}}\,. \tag{3.67}$$

JHEP11(1999)019

To solve this equation we will impose the conditions

$$\Gamma_{\underline{0}}\gamma_*\epsilon_0 = \epsilon_0\,, \qquad \Gamma_{\underline{05\natural}}\epsilon_0 = \epsilon_0\,, \tag{3.68}$$

which are the conditions expected for an $M5$-brane with an $M2$-brane ending on it. One can then deduce the following identities :

$$\begin{aligned} \Gamma_{\underline{0}}\gamma_*\epsilon &= \left(\cos\theta - \Gamma_{\underline{\natural\theta}}\sin\theta\right)\epsilon\,, \\ \Gamma_{\underline{05\natural}}\epsilon &= \left(\cos\theta - \Gamma_{\underline{\natural\theta}}\sin\theta\right)\epsilon\,. \end{aligned} \tag{3.69}$$

The matrix multiplying ϵ on the right hand side of (3.65) can be written as

$$\begin{aligned} &\Delta_4\left(r\sin\theta\right)' - \Pi^i\partial_i\left(r\cos\theta\right) + \Gamma_{\underline{\natural\theta}}\left[\Delta_4\left(r\cos\theta\right)' + \Pi^i\partial_i\left(r\sin\theta\right)\right] + \\ &\quad + \Gamma_{\underline{\natural}}\gamma_{\hat{\imath}}\left(\Delta_4\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\jmath}}r\cos\theta + \Pi^{\hat{\imath}}r\cos\theta\right) + \gamma_{\hat{\imath}}\Gamma_{\underline{\theta}}\left(\Delta_4\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\jmath}}r\sin\theta + \Pi^{\hat{\imath}}r\sin\theta\right). \end{aligned} \tag{3.70}$$

Requiring the coefficient of $\Gamma_{\underline{\natural}}\gamma_{\hat{\imath}}$ to vanish, we find that

$$\Pi^{\hat{\imath}} = -\Delta_4\frac{\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\jmath}}r}{r}\,. \tag{3.71}$$

Requiring the coefficient of $\Gamma_{\underline{\natural\theta}}$ to vanish and using (3.71) we then find that

$$\Pi^{\theta}(r\sin\theta)' = -\Delta_4\left[(r\cos\theta)' - \frac{\bar{g}^{\hat{\imath}\hat{\jmath}}\partial_{\hat{\imath}}r\partial_{\hat{\jmath}}r\sin\theta}{r}\right]. \tag{3.72}$$

Notice that (3.71) and (3.72) are entirely equivalent to the equations saturating the energy bound (3.27) and (3.28). It is straightforward to check that when (3.71) is satisfied, the coefficient of $\gamma_{\hat{\imath}}\Gamma_{\underline{\theta}}$ vanishes identically, so (3.70) is satisfied as a consequence of (3.71) and (3.72) provided that

$$U^{-1/3}\sqrt{\det\left(g_{ij}+K_iK_j\right)} = \frac{\Delta_4\,r^2}{(r\sin\theta)'}\left(1+\frac{\bar{g}^{ij}\partial_i r\partial_j r}{r^2}\right). \tag{3.73}$$

Since

$$\det\left(g_{ij}+K_iK_j\right) = (\det g)\left(1+g^{ij}K_iK_j\right) \tag{3.74}$$

one can use (3.66) to rewrite the left hand side of (3.73) as

$$U^{-1/3}\det g + U^{1/3}\left[r^2\Pi^i\Pi^j\bar{g}_{ij} + \left(\Pi^i\partial_i r\right)^2\right], \tag{3.75}$$

which can be shown to satisfy (3.73) when (3.71) and (3.72) are satisfied. We have thus shown that equations (3.71) and (3.72) are sufficient for preservation of 1/4 supersymmetry. As we saw earlier, when these equations are combined with the Bianchi identity they become equivalent to the second order equation (3.30). Any solution of this equation is therefore 1/4 supersymmetric.

JHEP11(1999)019

3.4 Multi-angle M5 solutions with 1/4 supersymmetry

We shall proceed as in section 2.3 to find more general solutions to equation (3.30) in the near horizon region. Our starting point will be the SO(4) invariant solution for $\nu \neq 0$, which is [13, 14] :

$$r_\nu(\theta) = C\frac{(\Lambda(\theta))^{1/2}}{\sin\theta}\,, \tag{3.76}$$

where $\Lambda(\theta) = 2(1-2\nu-\cos\theta)$. This function satisfies the following identity:

$$\frac{d}{d\theta}\left(\frac{r_\nu \sin^2\theta}{(r_\nu\sin\theta)'}\right) = 2\sin\theta\,. \tag{3.77}$$

Using the separation of variables ansatz

$$r(\theta,\theta_{\hat{\imath}}) = r_\nu(\theta)\prod_{\hat{\imath}}\delta_{\hat{\imath}}(\theta_{\hat{\imath}})\,, \tag{3.78}$$

we find that (3.30) reduces to

$$\begin{aligned} 0 = {} & \sin\theta\sin\theta_2\left[2\sin\theta_3\cos\theta_3\psi_3 + \sin^2\theta_3\psi_3'\right] + \\ & + \sin\theta\left[\cos\theta_2\psi_2 + \sin\theta_2\psi_2'\right] + \frac{\sin\theta}{\sin\theta_2}\psi_1' - \\ & - \sin^2\theta_3\sin\theta_2\frac{d}{d\theta}\left(\frac{r_\nu\sin^2\theta}{(r_\nu\sin\theta)'}\right)\left[\psi_3^2 + \frac{\psi_2^2}{\sin^2\theta_3} + \frac{\psi_1^2}{\sin^2\theta_3\sin^2\theta_2}\right], \end{aligned} \tag{3.79}$$

where $\psi_{\hat{\imath}}(\theta_{\hat{\imath}})$ are defined as in section 2.3. Using the identity (3.77), this can be shown to be equivalent to

$$\begin{aligned} 0 = {} & \sin\theta_2\left[2\sin\theta_3\cos\theta_3\psi_3 + \sin^2\theta_3\psi_3' - 2\sin^2\theta_3\psi_3^2\right] + \\ & + \left[\cos\theta_2\psi_2 + \sin\theta_2\psi_2' - 2\sin\theta_2\psi_2^2\right] + \\ & + \frac{1}{\sin\theta_2}\left[\psi_1' - 2\psi_1^2\right]. \end{aligned} \tag{3.80}$$

We can solve this equation as before by choosing the ψ-functions to satisfy the Bernoulli equations[5]

$$\psi_1' + 2\cot\theta_1\psi_1 = 2\psi_1^2\,, \tag{3.81}$$

$$\psi_2' + \cot\theta_2\psi_2 = 2\psi_2^2\,, \tag{3.82}$$

$$\psi_3' = 2\psi_3^2\,. \tag{3.83}$$

In addition to the trivial solution $\psi_{\hat{\imath}}(\theta_{\hat{\imath}}) = 0$, these have the non-trivial solutions

$$\frac{1}{\psi_1} = c_1\sin^2\theta_1 + 2\sin\theta_1\cos\theta_1\,, \tag{3.84}$$

$$\frac{1}{\psi_2} = c_2\sin\theta_2 - 2\sin\theta_2\log\tan\frac{\theta_2}{2}\,, \tag{3.85}$$

$$\frac{1}{\psi_3} = c_3 - 2\theta_3\,. \tag{3.86}$$

[5] Analogous generalizations to the ones pointed out in section 2.3 also apply in this case.

JHEP11(1999)019

The singularity structure of the new M5-brane solutions is similar to what we found earlier for the $D5$-brane. Any non-constant function $\delta_{\hat{\imath}}$ will have a branch cut singularities at some angle $\alpha_{\hat{\imath}}$ of the type

$$\delta_{\hat{\imath}} \sim \frac{1}{(\theta_{\hat{\imath}} - \alpha_{\hat{\imath}})^{1/2}} \,. \tag{3.87}$$

To compute the energy of the new solutions we would need the conjugate momenta to the angles. These are

$$\begin{aligned}
\Pi^3 &= -R^3 \sin\theta \sqrt{\det{}^3 g}\, \psi_3(\theta_3)\,, && (3.88)\\
\Pi^2 &= -R^3 \sin\theta \sqrt{\det{}^3 g}\, \frac{\psi_2(\theta_2)}{\sin^2\theta_3}\,, && (3.89)\\
\Pi^1 &= -R^3 \sin\theta \sqrt{\det{}^3 g}\, \frac{\psi_1(\theta_1)}{\sin^2\theta_3 \sin^2\theta_2}\,, && (3.90)\\
\Pi^\theta &= \Pi(\theta) \sqrt{\det{}^3 g} + 4R^3 \Lambda(\theta) \sqrt{\det{}^3 g}\, \times \\
&\qquad \times \left[(\psi_3(\theta_3))^2 + \frac{(\psi_2(\theta_2))^2}{\sin^2\theta_3} + \frac{(\psi(\theta_1))^2}{\sin^2\theta_3 \sin^2\theta_2} \right], && (3.91)
\end{aligned}$$

The corresponding topological charge is

$$\begin{aligned}
\mathcal{Z}_4 = &\left[\mathcal{Z}(\theta) \sqrt{\det{}^3 g} + 4R^3 \frac{r_0(\theta)}{\sin\theta} \Lambda(\theta) \sqrt{\det{}^3 g}\, \times \right.\\
&\left. \times \left((\psi_3(\theta_3))^2 + \frac{(\psi_2(\theta_2))^2}{\sin^2\theta_3} + \frac{(\psi(\theta_1))^2}{\sin^2\theta_3 \sin^2\theta_2} \right) \right] \prod_{\hat{\imath}} \delta_{\hat{\imath}}(\theta_{\hat{\imath}})\,. \qquad (3.92)
\end{aligned}$$

It can be verified that the Π's satisfy (3.17). It is again remarkable that the new contributions to the topological charge $\psi_{\hat{\imath}}$ are positive whenever the SO(4) invariant configuration is positive (for the full range of θ when $\nu = 0$). As for the D5-D3 configuration, $\mathcal{Z}_4$ can be rewritten as a divergence, $\mathcal{Z}_4 = \partial_i \mathcal{Z}_4^i$, at least in the near horizon limit. Taking this limit in (3.25) we find that

$$\begin{aligned}
\mathcal{Z}_4^\theta &= -\Pi^\theta\, r\cos\theta + R^3 \sin^4\theta \sqrt{\det{}^3 g}\, r \\
\mathcal{Z}_4^{\hat{\imath}} &= -\,\Pi^{\hat{\imath}}\, r\cos\theta\,. \qquad (3.93)
\end{aligned}$$

4. Discussion

A universal feature of supersymmetric p-branes is the appearance in the action governing their dynamics of a Wess-Zumino (WZ) term. In the simplest cases this term just describes the coupling of the brane to a $(p+1)$-form potential of the supergravity background, and the physics associated with this coupling has been extensively investigated in the past. For both D-branes and the M5-brane this coupling is only

JHEP11(1999)019

the first term in an expansion of the WZ term in powers of the field strength of a worldvolume gauge field. In certain backgrounds the first non-leading term in this expansion provides a source term for the worldvolume gauge field which, as a consequence, cannot vanish. The baryonic brane configurations discussed here, and in previous work, provide examples of the physics associated with this phenomenon. In previous work a 1/4 supersymmetric baryonic D5-brane with SO(5) symmetry was found and shown to saturate an energy bound, and similar results have been established for other D-branes. One result of this paper is an extension of these results to the M5-brane. In this case the baryonic brane provides a worldvolume realization of the baryon string-vertex of the $(2,0)$ superconformal field theory which, according to the AdS/CFT correspondence, lives on the boundary of the $AdS_4 \times S^7$ near-horizon background of the supergravity M5-brane.

Another purpose of this paper has been to find the *general* conditions implied either by saturation of a Bogomol'nyi-type energy bound or by 1/4 supersymmetry, for both baryonic D5-branes and baryonic M5-branes. We have found these conditions, and we have found that they coincide, i.e. configurations saturating the bound are 1/4 supersymmetric and vice-versa. At least, this is formally true in that both conditions yield the same second-order partial differential equation for a function r on S^5 (in the D5-brane case) or S^4 (in the M5-brane case). Of course, the equations for the two cases, D5 and M5 are different, but they are broadly similar. Particular solutions are easily found in the D5-brane case via an SO(5)-invariant ansatz. The condition for 1/4 supersymmetry then reduces to one found previously, and the preciously found baryonic brane solution is recovered. We have shown how a similar SO(4)-invariant ansatz yields an explicit baryonic M5-brane solution.

In going beyond these simple solutions of high symmetry one runs into the difficulty that a given local solution of the 1/4 supersymmetry condition will not generally be well defined on the 5-sphere (4-sphere in the M5 case, but we concentrate on the D5 case in the following discussion). Of course, no solution can really be well-defined on the 5-sphere because the D5-brane carries a non-zero charge but, for reasons that we have explained in the introduction, one might suppose that the singularities could be confined to points, which could then be interpreted as the endpoints of N strings. The SO(5)-invariant solution has a single point singularity which has been interpreted as the coincident endpoints of N parallel strings. However, although we have found many new 1/4 supersymmetric solutions, none of them has singularities of this type; in fact they have branch cuts. We suspect that this is due to the restricted nature of the ansatz that we have used to find these solutions. The full equation for 1/4 supersymmetry is not at all simple and one cannot expect to find all its solutions explicitly. Thus, our inability to find solutions with specified singularities is in no way an indication that they do not exist, although it is also true that we cannot prove that they do. The point is an important one because it has implications for potential limitations of the worldvolume approach to spacetime physics, but its full

JHEP11(1999)019

resolution must await future studies.

Acknowledgments

PKT thanks the ECM of the University of Barcelona for hospitality. J.S. thanks the ITP of the University of Hannover for hospitality. JS is supported by a fellowship from Comissionat per a Universitats i Recerca de la Generalitat de Catalunya. This work was supported in part by AEN98-0431 (CICYT), GC 1998SGR (CIRIT). J.G. and J.S. would like to thank J. Solà-Morales for useful discussions. AVR is grateful to J.M. Sanchez de Santos for discussions and early collaboration on some of the topics studied in this paper. The work of AVR is supported in part by CICYT under grant AEN96-1673 and by DGICYT under grant PB96-0960 and by the European Union grant ERBFM-RXCT960012.

References

[1] G.W. Gibbons and P.K. Townsend, *Vacuum interpolation in supergravity via super p-branes*, *Phys. Rev. Lett.* **71** (1993) 3754 [hep-th/9307049];
M.J. Duff, G.W. Gibbons, and P.K. Townsend, *Macroscopic superstrings as interpolating solitons*, *Phys. Lett.* **B 332** (1994) 321 [hep-th/9405124];
G.W. Gibbons, G.T. Horowitz and P.K. Townsend, *Higher-dimensional resolution of dilatonic black hole singularities*, *Class. and Quant. Grav.* **12** (1995) 297 [hep-th/9410073].

[2] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231, [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[3] A. Hanany and E. Witten, *Type-IIB superstrings, BPS monopoles and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[4] S.P. de Alwis, *Matrix models and string world sheet duality*, *Phys. Lett.* **B 423** (1998) 59 [hep-th/9710219].

[5] E. Witten, *Baryons and branes in anti-de Sitter space*, *J. High Energy Phys.* **07** (1998) 006 [hep-th/9805112].

[6] D.J. Gross and H. Ooguri, *Aspects of large-N gauge theory dynamics as seen by string theory*, *Phys. Rev.* **D 58** (1998) 106002 [hep-th/9805129].

[7] Y. Imamura, *Supersymmetries and BPS configurations on anti-de Sitter space*, *Nucl. Phys.* **B 537** (1999) 184 [hep-th/9807179].

JHEP11(1999)019

[8] C.G. Callan, A. Guijosa and K.G. Savvidy, *Baryons and string creation from the five-brane world volume action*, *Nucl. Phys.* **B 547** (1999) 127 [hep-th/9810092].

[9] B. Craps, J. Gomis, D. Mateos and A.V. Proeyen, *BPS solutions of a D5-brane world volume in a D3-brane background from superalgebras*, *J. High Energy Phys.* **04** (1999) 004 [hep-th/9901060].

[10] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The dirichlet super p-branes in ten-dimensional type-IIA and -IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159];
E. Bergshoeff and P. K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173];
M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249];
Gauge-invariant and gauge-fixed D-brane actions, *Nucl. Phys.* **B 495** (1997) 99.

[11] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158];
E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, *κ symmetry, supersymmetry and intersecting branes*, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040];
E. Bergshoeff and P. K. Townsend, *Solitons on the supermembrane*, *J. High Energy Phys.* **05** (1999) 021 [hep-th/9904020].

[12] M. Alishahiha, *$(0,2)$ theory and string baryon in M-theory on $AdS_7 \times S^4$*, hep-th/9811042.

[13] C.G. Callan, A. Guijosa, K.G. Savvidy and O. Tafjord, *Baryons and flux tubes in confining gauge theories from brane actions*, *Nucl. Phys.* **B 555** (1999) 183 [hep-th/9902197].

[14] J.M. Camino, A.V. Ramallo and J.M.S. de Santos, *World volume dynamics of D-branes in a D-brane background*, hep-th/9905118.

[15] E. Bergshoeff and P. K. Townsend, *Super D-branes revisited*, *Nucl. Phys.* **B 531** (1998) 226 [hep-th/9804011].

[16] J.P. Gauntlett, J. Gomis and P.K. Townsend, *BPS bounds for world volume branes*, *J. High Energy Phys.* **01** (1998) 003 [hep-th/9711205].

[17] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Baryons from supergravity*, *J. High Energy Phys.* **07** (1998) 020 [hep-th/9806158];
Y. Imamura, *Baryon mass and phase transitions in large-N gauge theory*, *Prog. Theor. Phys.* **100** (1998) 1263 [hep-th/9806162].

[18] H. Lu, C. N. Pope and J. Rahmfeld, *A construction of Killing spinors on S^n*, hep-th/9805151.

JHEP11(1999)019

[19] I. Bandos et al., *Covariant action for the superfive-brane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149].

[20] D. Sorokin, *On some features of the M5-brane*, hep-th/9807050.

[21] E. Bergshoeff, D. Sorokin and P.K. Townsend, *The M5-brane hamiltonian*, *Nucl. Phys.* **B 533** (1998) 303 [hep-th/9805065].

[22] P.S. Howe, N.D. Lambert and P.C. West, *The selfdual string soliton*, *Nucl. Phys.* **B 515** (1998) 203 [hep-th/9709014].

JHEP11(1999)019